# Efficient Estimation of Residual Trajectory Deviations from SAR Data

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Abstract—Insufficient accuracy of trajectory measurements is a pressing problem for modern high-resolution airborne SAR systems. In the paper, a novel method for the estimation of residual trajectory deviations from SAR data is proposed. The method is based on the map-drift autofocus principle used to estimate the cross-track components of the aircraft acceleration on short time intervals. The estimated acceleration is then integrated to retrieve the residual trajectory deviations on the whole data frame. The proposed approach has been successfully tested with an X-band airborne SAR system.

Keywords—synthetic aperture radar; trajectory deviations; motion error compensation; autofocus; map-drift.

#### I. INTRODUCTION

The quality of synthetic aperture radar (SAR) images is strongly related to the accuracy of the SAR platform trajectory measurements [1]-[3]. With a fast growth of spatial resolution of modern SAR systems the requirements to the measurement precision have become very high. Conventional navigation systems often do not provide the required accuracy. As the result, uncompensated trajectory deviations lead to residual phase errors in the processed data and deteriorate the image quality. Autofocus methods are used in such situations for precise estimation of the errors directly from the SAR data.

Depending on the phase error estimation principle, the autofocus methods are divided into two general classes – parametric and nonparametric methods. The parametric autofocus methods use some model for the platform motion. The aim is to estimate parameters of the model used. The most known parametric method is the map-drift autofocus (MDA) [4]-[8]. It is used for the estimation of a quadratic phase error by measuring the linear shift between two SAR images built by dividing the processing interval in the azimuth on two parts. The multiple-aperture map-drift autofocus [3], [9] estimates the phase error as a higher-order polynomial. Other models, such as a Fourier series, have also been proposed [9]-[11]. The main problem is that the parametric autofocus methods are restricted by the models applied.

The nonparametric methods do not use approximation models for phase errors. The most popular nonparametric technique is the phase gradient autofocus (PGA) [3], [12], [13], which estimates the first derivative (the gradient) of the phase error from backscattered signals of isolated point targets. However, the PGA is intended mainly for the spotlight SAR mode. In this SAR mode, the signals from all point targets on

the scene are presented in each radar pulse. Therefore, the motion error function can be estimated for the whole data acquisition interval. Unlike the spotlight mode, in the strip-map mode, the point target echo is presented in the received data only during the time required for the target to cross the antenna beam. It means that the signal of the selected point target contains only a small part of the complete motion error function.

In this paper, a new approach to the strip-map SAR autofocus is presented. The method is called "Local-Quadratic Map-Drift Autofocus" (LQMDA). The idea is to estimate the local quadratic phase errors on small time intervals. These local estimates are used to evaluate the cross-track acceleration components. The residual trajectory deviations are then obtained by double integration of the estimated acceleration. The proposed LQMDA has some similarities with the reflectivity displacement method (RDM) [14]-[16]. Both methods use local estimates of the motion errors and the integration steps to evaluate the complete phase error function.

In Section II, the principle of the representation of an arbitrary phase error by local approximations is described. The formation of SAR images and the application of the map-drift principle on short time intervals are considered in Section III. Section IV describes how the residual trajectory deviations can be derived from the local quadratic phase error estimates. Experimental results are discussed in Section V.

### II. PHASE ERRORS ON SHORT TIME INTERVALS

A precise SAR focusing requires the application of a motion error compensation procedure, which is based on accurate trajectory measurements. According to this technique [2], the difference between the expected straight line trajectory and the actual measured trajectory is compensated in the SAR data via the correction of the range migration errors  $\Delta R_{E}(R, t)$  and the corresponding compensation of the phase  $(4\pi/\lambda)\Delta R_E(R,t)$ . Here R is the slant range, t is the azimuth time, and  $\lambda$  is the radar wavelength. If the actual flight trajectory is measured accurately, the motion error compensation procedure prevents the degradation of the SAR image quality. However, in the case of a high resolution imaging, most of navigation systems cannot provide the required accuracy, and residual trajectory deviations are left uncompensated. These residual errors have to be estimated by autofocusing.

Let us consider the range-compressed backscattered signal s(R,t) after the application of both the motion error compensation and the range cell migration correction. It means that the signal is ready for the azimuth compression; however, it contains the residual phase errors  $\varphi_E(R,t)$ :

$$s(R, t) = s_{ref}(R, t) \exp[i\varphi_E(R, t)].$$

Here  $s_{ref}(R,t)$  is the signal expected for the reference flight conditions. Let us divide the whole data frame interval  $0 \le t \le T_{FR}$  of the duration  $T_{FR}$  on half-overlapped short-time intervals of the duration  $T_s$ . Assuming that these intervals are short enough, the residual phase error within each interval can be approximated by the 2-nd order polynomial:

$$\varphi_E(R, t_n + \tau) \approx \varphi_E(R, t_n) + \varphi_E'(R, t_n)\tau + \varphi_E''(R, t_n)\tau^2 / 2$$
.

Here  $\tau$  is the time within the short interval  $-T_s / 2 < \tau < T_s / 2$ .

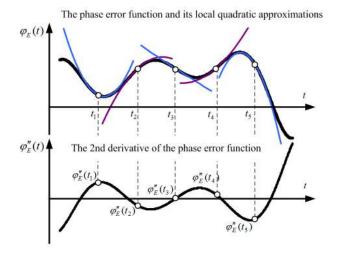


Fig. 1. The residual phase error function (on a given range), its quadratic approximations, and the 2-nd derivative of the error function.

The described local representation can be effectively used for the estimation of an arbitrary residual phase error function. We propose to estimate the local quadratic error coefficients  $\varphi_E''(R, t_n)$  on each short internal separately by using the mapdrift principle. According to this autofocus technique, the short processing interval is divided on two parts, and two SAR images are built. The presence of the quadratic error shifts the images in the azimuth in the opposite directions. By measuring this shift, the local quadratic phase error coefficient  $\varphi_{\scriptscriptstyle E}''(R,t_{\scriptscriptstyle \perp})$ can be estimated. The constant term  $\varphi_{E}(R, t_{n})$  does not affect the estimation and can be omitted, as well as the linear term  $\varphi_E'(R,t_n)\tau$  that only shifts the two SAR images in the azimuth in the same direction. As the result, we can estimate the samples of the second derivative of the residual error function as illustrated in Fig. 1. Next, by double integration, we can evaluate the unknown residual phase error function for the whole data frame and use it for the compensation. We suggested this approach in [17].

However, the problem is that the motion errors in the data depend on range. It means that the phase error functions have to be estimated for each range gate. In order to take into account the range dependence, in this paper we propose to estimate the cross-track components of the aircraft acceleration,  $a_{y}(t_{n})$  and  $a_{z}(t_{n})$ , which are responsible for the local quadratic phase errors and related to the estimated values  $\varphi_E''(R, t_n)$ . We expect that the along-track component of the acceleration is measured more accurately by the SAR navigation system than the cross-track components. Therefore, the residual (uncompensated) along-track acceleration  $a_{\nu}(t_{\nu})$ is assumed to be negligibly small compared to the cross-track ones. By double integration of the estimated acceleration, the residual trajectory deviations can be evaluated. Then, the range-dependent phase errors can be calculated and used for the motion error compensation.

# III. SAR PROCESSING ON SHORT TIME INTERVALS AND ESTIMATION OF LOCAL QUADRATIC PHASE ERRORS

SAR data collected on a short time interval can be processed with the range-Doppler algorithm [18] adopted for the application on the short time interval [17] or with the dechirp algorithm [3], [18]. The SAR image formed on the short time interval shows the part of the scene illuminated by the real antenna beam during this interval. For autofocusing, the duration of the short time intervals  $T_s$  is assumed to be much smaller than the time required for the target to cross the real antenna footprint. In this case, the illuminated part of the scene is approximately equal to the antenna footprint area. For example, for the antenna beam width  $\theta_A = 10^{\circ}$  in the azimuth, the antenna footprint on the ground at the slant range R = 4km is  $L_A \approx R\theta_A \approx 700$  m. The synthetic aperture length required to achieve the azimuth resolution  $\rho = 3$  m is only  $L_s \approx K_w \lambda R/(2\rho) \approx 26$  m (  $\lambda = 3$  cm and the weighting window coefficient  $K_w = 1.3$ ). In Fig. 2, one can see antenna footprints on the ground plane for three positions of the aircraft: 1) at the beginning, 2) at the center, and 3) at the end of the short interval. SAR images, which are built on each half of the short-time interval (for autofocusing), represent two highly overlapped antenna footprints.

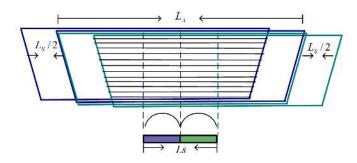


Fig. 2. Radar data in the time domain and the antenna footprints.

In the presence of a local quadratic phase error there will be a linear shift  $\Delta t_{\text{max}}$  between the two SAR images, which can be

easily measured from the position of the maximum of the correlation function. The local phase error coefficient is

$$\varphi_E''(R, t_n) = 2\pi F_{DR}^E(R, t_n) = 2\pi F_{DR}(R) \Delta t_{\text{max}} / (T_S / 2),$$

where  $F_{DR}$  is the Doppler rate of the SAR matched filter.

## IV. ESTIMATION OF TRAJECTORY DEVIATIONS FROM LOCAL QUADRATIC PHASE ERRORS

Uncompensated trajectory deviations lead to rangedependent phase errors in SAR data. This effect is especially significant in the cases of low-altitude, wide-swath or squinted SAR modes. In order to account for the range dependence, we propose to derive the residual trajectory deviations from the estimated local quadratic phase errors.

Let us approximate the aircraft trajectory on the short time interval as follows:

$$\vec{\mathbf{r}}_{A}(t_{n}+\tau) = \vec{\mathbf{H}} + \vec{\mathbf{V}}(t_{n}+\tau) + \vec{\mathbf{r}}_{E}(t_{n}+\tau) ,$$

$$\vec{\mathbf{r}}_{E}(t_{n}+\tau) \approx \vec{\mathbf{r}}(t_{n}) + \vec{\mathbf{v}}(t_{n})(t_{n}+\tau) + \vec{\mathbf{a}}(t_{n})(t_{n}+\tau)^{2}/2 .$$

Here  $\vec{\mathbf{r}}_{E}(t_{n} + \tau)$  represents the residual trajectory deviations;  $\vec{\mathbf{H}} = (0, 0, H)$  and  $\vec{\mathbf{V}} = (V, 0, 0)$  are the reference flight altitude and velocity. The Doppler rate of the backscattered signal under such motion, assuming that  $|\vec{\mathbf{r}}_{E}(t_{n})| << R$ , is

$$F_{DR}(R, t_n) \approx \frac{2}{\lambda} \left[ \frac{1}{R} \left( |\vec{\mathbf{V}} + \vec{\mathbf{v}}(t_n)|^2 - \left( \frac{\vec{\mathbf{R}} \cdot (\vec{\mathbf{V}} + \vec{\mathbf{v}}(t_n))}{R} \right)^2 \right) - \frac{\vec{\mathbf{R}} \cdot \vec{\mathbf{a}}(t_n)}{R} \right],$$

where  $\vec{\mathbf{R}} = (x_R, y_R, -H)$  is the slant range vector, and

$$x_R = H \tan \alpha \cos \beta + \sin \beta \sqrt{R^2 - H^2 - (H \tan \alpha)^2}$$
,

$$y_R = -H \tan \alpha \sin \beta + \cos \beta \sqrt{R^2 - H^2 - (H \tan \alpha)^2},$$

are the coordinates of the point on the Doppler centroid line at the range R;  $\alpha$  and  $\beta$  are the antenna beam pitch and yaw angles.

The estimated local quadratic phase errors are related to the Doppler rate errors. The main contribution to these errors is introduced by the uncompensated cross-track acceleration:

$$F_{DR}^{E}(R,t_{n}) \approx \frac{2}{\lambda} \frac{\vec{\mathbf{R}} \cdot \vec{\mathbf{a}}(t_{n})}{R} \approx \frac{2}{\lambda} \frac{y_{R} a_{Y}(t_{n}) - H a_{Z}(t_{n})}{R},$$

The uncompensated velocity  $\vec{\mathbf{v}}(t_n)$  has a minor influence; it is accounted implicitly during iterations of the autofocus procedure.

From the above formula one can easily write a system of linear equations that provide the mean square error (MSE) estimates for the acceleration components:

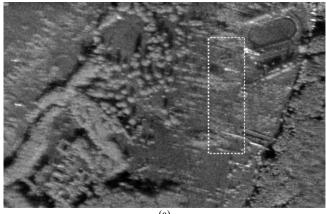
$$\begin{split} &=\sum_{m=1}^{M_R} \left[ \frac{2}{\lambda} \frac{y_{_m} a_{_Y}(t_{_n}) - H a_{_Z}(t_{_n})}{R_{_m}} - F_{_{DR}}^{_E}(R_{_m},t_{_n}) \right]^2, \\ &a_{_Y}(t_{_n}) \sum_{m=1}^{M_R} \left( \frac{y_{_m}}{R_{_m}} \right)^2 - a_{_Z}(t_{_n}) \sum_{m=1}^{M_R} \left( \frac{y_{_m}}{R_{_m}} \frac{H}{R_{_m}} \right) = \sum_{m=1}^{M_R} \frac{\lambda}{2} F_{_{DR}}^{_E}(R_{_m},t_{_n}) \frac{y_{_m}}{R_{_m}}, \\ &a_{_Y}(t_{_n}) \sum_{m=1}^{M_R} \left( \frac{y_{_m}}{R_{_m}} \frac{H}{R_{_m}} \right) - a_{_Z}(t_{_n}) \sum_{m=1}^{M_R} \left( \frac{H}{R_{_m}} \right)^2 = \sum_{m=1}^{M_R} \frac{\lambda}{2} F_{_{DR}}^{_E}(R_{_m},t_{_n}) \frac{H}{R_{_m}}. \end{split}$$

 $MSE(a_v(t_n), a_z(t_n)) =$ 

By solving this system independently for each short time interval with the centers at  $t_n$ , the sequence of the estimated acceleration  $a_{\scriptscriptstyle Y}(t_n), a_{\scriptscriptstyle Z}(t_n)$  is obtained. The residual trajectory deviations are calculated by double integration and used for the motion compensation. The described autofocusing procedure should be iterated several times for the best result.

### V. EXPERIMENTAL RESULTS

In this section, the performance of the proposed LQMDA method is illustrated with real data obtained with the airborne RIAN-SAR-X system [19], [20]. This SAR system has been developed and produced at the Institute of Radio Astronomy of the National Academy of Sciences of Ukraine.



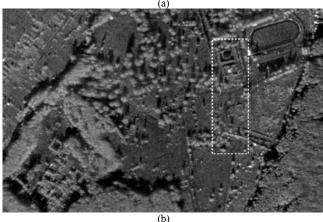


Fig. 3. 25 looks, 3-meter resolution SAR images built without autofocusing (a) and with the proposed LQMDA algorithm (b).

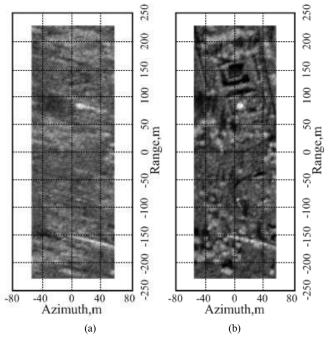


Fig. 4. The fragments of the SAR images shown in Fig. 3: (a) built without autofocusing, (b) built with the proposed LQMDA algorithm.

A 25-look SAR image with a 3-meter resolution built without autofocusing is shown in Fig 3a. One can observe non-uniform defocusing of the image. It means that the residual phase errors are time-varying. The well-focused SAR image built with the proposed LQMDA algorithm is shown in Fig. 3b. One can see that the proposed approach significantly improves the image quality. The influence of motion errors is evidently seen on small objects in the SAR images. Figure 4 shows the fragments of the scene taken from the SAR images in Fig. 3.

The provided example demonstrates the high performance of the LQMDA method, its ability to handle time-varying and range-dependent phase errors and evaluate uncompensated residual trajectory deviations.

### VI. CONCLUSION

In this paper, an efficient local-quadratic map-drift autofocus (LQMDA) method for stripmap SAR systems has been developed and tested. The approach enables for the evaluation of the residual trajectory deviations by the estimation of the cross-track components of the aircraft acceleration (followed by double integration). It should be emphasized that the method can handle time-varying and range-dependent phase errors. The high efficiency of the method has been proved by the real data processing.

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